Typed Mobile Ambients in Maude*

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Abstract. Maude has revealed as a powerful tool for implementing different kinds of semantics so that quick prototypes are available for trying examples and proving properties. In this paper we show how to define in Maude two semantics for Cardelli's Ambient Calculus. The first one is the operational (reduction) semantics which requires the definition of Maude strategies in order to avoid infinite loops. The second one is a type system defined by Cardelli to avoid communication errors. The correctness of that system was not formally proved. We enrich the operational semantics with error rules and prove that well-typed processes do not produce such errors. The type system is highly non-deterministic. We provide two different (equivalent) ways of implementing such non-determinism in the rules.

Keywords: Ambient calculus, operational semantics, type systems, Maude.

1 Introduction

Maude, a high-level language and high-performance system supporting both equational and rewriting logic computation [11, 10], has revealed as a powerful tool for representing different kinds of semantics [13, 22, 23]. Since Maude specifications are executable, what we get is an implementation of the language so that quick prototypes are available for trying examples and proving properties.

Among the advantages of rewriting logic (and Maude), we may emphasize the following:

– It is a simple formalism, with only a few rules of deduction that are easy to understand and justify.
– It is very flexible and expressive, capable of representing change in systems with very different structure.
– It allows user-definable syntax, with complete freedom to choose the operators and structural properties appropriate for each problem.
– It is intrinsically concurrent, representing concurrent change and supporting reasoning about such change.
– It has initial models, that can be intuitively understood as providing "no junk" and "no confusion."
– It is realizable in the wide spectrum logical language Maude, supporting executable specification and programming.

We use Maude as a *metalanguage* [9] in which the syntax and semantics of particular languages can be formally defined. One of our aims is to maintain the representation distance as short as possible. There are several different ways of mapping inference systems into rewriting logic. In the structural operational semantics case, judgements typically

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have the form of some kind of transition $P \to Q$ between states so that it makes sense to consider the possibility of mapping directly this transition relation between states to a rewriting relation between terms representing the states. When thinking this way, an inference rule of the form

$$\frac{P_1 \to Q_1 \quad \dots \quad P_n \to Q_n}{P_0 \to Q_0}$$

becomes a *conditional* rewrite rule of the form

$$P_0 \longrightarrow Q_0 \quad \textit{if} \quad P_1 \longrightarrow Q_1 \wedge \dots \wedge P_n \longrightarrow Q_n,$$

where the condition includes rewrites. In this way the semantic rules become (conditional) rewrite rules, where the transition in the conclusion becomes the main rewrite of the rule, and the transitions in the premises become rewrite conditions.

In this paper we show how to define in Maude two semantics for Cardelli's Ambient Calculus (AC). First, we define the operational semantics given by Cardelli as structural congruence and reduction rules. We show how to exploit the rewriting machinery in order to reduce the number of rules, and also the decisions we have taken in order to avoid infinite reductions. Rewrite rules need not be confluent or terminating. This theoretical generality needs some control when the specifications become executable, because the user needs to make sure that the rewriting process does not go in undesired directions. We have recently defined a strategy language for Maude, to control the rewriting process [14]. Strategy expressions can be defined to reduce the tree of rewritings of a given term. For example, replication in AC requires the definition of strategies to avoid infinite loops.

Cardelli defined several type systems for AC [7, 5, 6] in order to avoid different kinds of errors: basically communication errors, and violation of mobility and opening constraints. We study here how to implement the first type system defined by Cardelli [7] to detect communication errors, and also syntactically incorrect terms. There, Cardelli proved a subject reduction theorem in order to justify the correctness of the type system, but only an intuitive meaning of types was given. It is an interesting exercise to formalize it in order to complete the correctness proof. We enrich the operational semantics with error reductions reflecting the kind of errors we want to avoid. Then we prove (by hand) that a well-typed process never causes such an error. We implement these error rules in Maude and also the type system.

The type rules are highly nondeterministic. We provide two (equivalent) ways of managing nondeterminism to implement the rules and discuss their advantages and disadvantages. Both implementations infer the process type as the result of the rewriting.

Additionally, as a consequence of the study of the type rules we have encountered that adding a new rule, more processes that do not produce communication errors can be typed, and consequently we have slightly increased the power of the type system.

The representation in Maude of the AC operational semantics and type systems, in a way quite close to the original mathematical formulation, has provided us interpreters where these inference systems can be executed. Our final aim is to go one step further in the exploitation of using Maude and take advantage of the formal tools designed for this language. For example, automatic reasoning about specifications in Maude is supported by the experimental ITP tool [8], a rewriting-based theorem prover (implemented also in Maude) that can be used to prove inductive properties of equational specifications.

The rest of the paper is organized as follows. Section 2 presents a short description of Maude and the strategy language we use. Section 3 reviews the ambient calculus briefly. Section 4 describes the implementation of the operational semantics of AC and uses it to execute a non-trivial example. Section 5 enriches the semantics with error rules and implements the type system for avoiding communication errors. Finally Section 6 gives conclusions and future work.

2 Maude in a Nutshell

In rewriting logic and Maude the data on the one hand and the state of a system on the other are both formally specified as an algebraic data type by means of an equational specification. Maude uses a very expressive version of equational logic, namely *membership equational logic* [1]. In this kind of specifications we can define new types (by means of keyword `sort(s)`); subtype relations (understood as inclusion relations) between types (`subsort`); operators (`op`) for building values of these types, giving the types of their arguments and result, and which may have attributes as being associative (`assoc`) or commutative (`comm`), for example; equations (`eq`) that identify terms built with these operators; and memberships (`mb`) $t : s$ stating that the term t has sort s. Both equations and memberships can be conditional, with respective keywords `ceq` and `cmb`. Conditions are formed by a conjunction (written `/\`) of equations and memberships. The following *functional* module (with syntax `fmod...endfm`) defines the syntax of a vending machine where apples (`a`) and cakes (`c`) can be bought by using dollars (`$`) and quarters (`q`):

```
fmod VENDING-MACHINE-SIGNATURE is
  sorts Coin Item Marking .
  subsorts Coin Item < Marking .
  op __ : Marking Marking -> Marking [assoc comm id: null] .
  op null : -> Marking .
  ops $ q : -> Coin .
  ops a c : -> Item .
endfm
```

Equations are assumed to be confluent and terminating, that is, we can use the equations from left to right to reduce a term t to a unique, canonical form t' (modulo the operators attributes as associativity, commutativity, and identity) that is equivalent to t (they represent the same value).

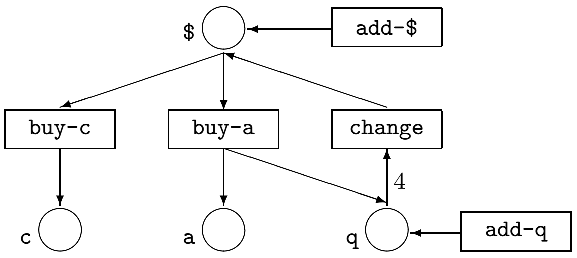
The *dynamic* behavior of a system is specified by rewrite rules of the form

$$t \longrightarrow t' \quad \textit{if} \quad (\bigwedge_i u_i = v_i) \wedge (\bigwedge_j w_j : s_j) \wedge (\bigwedge_k p_k \longrightarrow q_k)$$

that describe the local, concurrent transitions of the system. That is, when a part of a system matches the pattern t and the conditions are fulfilled, it can be transformed into the corresponding instance of the pattern t'. Rewrite rules are included in *system* modules (with syntax `mod...endm`).

For example, the next module specifies a machine to buy cakes and apples with dollars and quarters, which can be represented graphically as shown on the right. A cake costs a dollar and an apple three quarters. We can insert dollars and quarters in the machine, although due to an unfortunate design, the machine only accepts buying cakes and apples with dollars. When the user buys an apple the machine takes a dollar and returns a quarter. The machine can also change four quarters into a dollar.

```
mod VENDING-MACHINE is
  inc VENDING-MACHINE-SIGNATURE .
  var M : Marking .
  rl [add-q]  : M => M q .
  rl [add-$]  : M => M $ .
  rl [buy-c]  : $ => c .
  rl [buy-a]  : $ => a q .
  rl [change]: q q q q => $ .
endm
```



2.1 Strategies

Because system modules are rewrite theories that do not need to be neither confluent nor terminating (as the previous example shows), we need to have good ways of controlling the rewriting inference process—which in principle could not terminate or may go in many undesired directions—by means of adequate *strategies*. We have defined a strategy language for Maude that can be used to control how rules are applied to rewrite a term [14]. The simplest strategies are the constants `idle`, which always succeeds by doing nothing, and `fail`, which always fails. The basic strategies consist of the application of a rule (identified by the corresponding rule label) to a given term. In this case a rule is applied *anywhere* in the term where it matches satisfying its condition. When the rule being applied is a conditional rule with rewrites in the conditions, the strategy language allows to control by means of search expressions how the rewrite conditions are solved. An operation `top` to restrict the application of a rule just to the *top* of the term is also provided. Basic strategies are then combined so that strategies are applied to execution paths. Some strategy combinators are the typical regular expression constructions: concatenation (`;`), union (`|`), and iteration (`*` for 0 or more iterations, `+` for 1 or more, and `!` for a "repeat until the end" iteration). Another strategy combinator is a typical if-then-else, but generalized so that the first argument is also a strategy. The language also provide an `orelse` combinator where the second strategy is applied only when the first one is unsuccessful, and a (x)`matchrew` combinator that allows a term to be split in subterms, and specifies how these subterms have to be rewritten.

In the vending machine example, if we start with two dollars and want to buy a cake and an apple, we use the strategy `buy-c ; buy-a`. If we want to spend all our money buying apples, we use the strategy `(buy-a ! ; change !) !` where we also use the rule `change` in order to obtain dollars from four quarters.

3 Cardelli's Ambient Calculus

In this section we will present the basic notions about the Ambient Calculus [4], a process algebra that focuses on the notions of locations, mobility (of agents and their environments) and authorizations (to move or interact). *Ambients* will be the main entities of this model. An ambient is a place limited by a boundary where computations take place. They are hierarchically structured, so that we do not abstract from the path needed to arrive at the destination. Agents are confined to ambients and ambients move under the control of agents, allowing the movement of nested environments, that also include data and live computation.

In Figure 1 the syntax for AC with communication primitives is presented. We will consider two disjoint sets, $\mathcal{N} = \{m, n, \ldots\}$ for names and $\mathit{Var} = \{x, y, \ldots\}$ for variables, and a special symbol ϵ. We will denote $\mathit{Id} = \mathcal{N} \cup \mathit{Var}$ and $\mathit{Cap} = \{\mathit{in}\ N, \mathit{out}\ N, \mathit{open}\ N \mid N \in \mathit{Id}\}$. We will denote as A^* the set of paths (sequences) formed over elements of A.

Processes

$P, Q ::=$	processes
$(\nu n : W)P$	restriction
0	inactivity
$P \mid Q$	composition
$!P$	replication
$M[P]$	ambient
$M.P$	capability action
$(x_1 : W_1, \ldots, x_n : W_n)P$	input action
$\langle M_1, \ldots, M_n \rangle$	asynchronous output action

Expressions

$M ::=$	capabilities
x	variable
n	name
$in\ M$	can enter into M
$out\ M$	can exit out of M
$open\ M$	can open M
ϵ	null
$M.M'$	path

Fig. 1. Syntax of the Ambient Calculus

Ambients are denoted as $n[P]$, where n is its name and P is its content, which is essentially a parallel composition of sequential processes and subambients. These sequential processes can be prefixed processes, $M.P$, meaning that it must consume M before proceeding with P, polyadic inputs $(x_1 : W_1, \ldots, x_n : W_n)P$ and polyadic asynchronous outputs $\langle M_1, \ldots, M_n \rangle$. We will suppose that the variables appearing in the input construction are pairwise distinct. Also, new names can be created (restriction) $(\nu n : W)P$ and processes may be replicated $!P$. There is a special process 0 that is inactive.

Notice that for simplicity, in the syntax definition ambient names and capabilities belong to the same syntactic category. As a consequence the syntax allows the construction of meaningless processes such as $n.P$ or $in\ n[P]$. Later these terms will be ruled out by the type system that we will discuss in Section 5.

The operational semantics of the language is defined by means of a structural congruence relation $\equiv$ and a reduction relation $\rightarrow$. The former basically identifies those processes that are equivalent up to some trivial syntactic reorganization. It is the least equivalence relation satisfying the rules in Figure 2. For example, the inactive process 0 can be eliminated (or added) when in parallel with other processes.

In addition, processes are identified by α-conversion up to the renaming of bound names and variables:

$$(\nu n : W)P = (\nu m : W)P\{n := m\} \quad \textit{if } m \notin \textit{fn}(P)$$

$$(x_1, \ldots, x_n)P = (y_1, \ldots, y_n)P\{x_i := y_i\} \quad \textit{if } y_i \notin \textit{fv}(P)$$

A restricted name cannot be used outside its scope. However, α-conversion can be used to avoid name clashes, and in this way it is reflected the fact that the restricted name cannot be known, in principle, out of the restricted term. By means of the extrusion rule we can augment the scope of the restriction from a parallel component to the whole parallel

Structural congruence

$P \equiv Q \Rightarrow (\nu n)P \equiv (\nu n)Q$	(Struct Res)
$P \equiv Q \Rightarrow P \mid R \equiv Q \mid R$	(Struct Par)
$P \equiv Q \Rightarrow !P \equiv !Q$	(Struct Repl)
$P \equiv Q \Rightarrow n[P] \equiv n[Q]$	(Struct Amb)
$P \equiv Q \Rightarrow M.P \equiv M.Q$	(Struct Action)
$P \mid Q \equiv Q \mid P$	(Struct Par Comm)
$(P \mid Q) \mid R \equiv P \mid (Q \mid R)$	(Struct Par Assoc)
$!P \equiv P \mid !P$	(Struct Repl Par)
$(\nu n)(\nu m)P \equiv (\nu m)(\nu n)P$	(Struct Res Res)
$(\nu n)(P \mid Q) \equiv P \mid (\nu n)Q$ if $n \notin fn(P)$	(Struct Res Par)
$(\nu n)(m[P]) \equiv m[(\nu n)P]$ if $n \neq m$	(Struct Res Amb)
$P \mid 0 \equiv P$	(Struct Zero Par)
$(\nu n)0 \equiv 0$	(Struct Zero Res)
$!0 \equiv 0$	(Struct Zero Repl)
$P \equiv Q \Rightarrow (x_1, \ldots, x_n)P \equiv (x_1, \ldots, x_n)Q$	(Struct Input)
$\epsilon.P \equiv P$	(Struct ϵ)
$(M.N).P \equiv M.(N.P)$	(Struct Path)

Reduction

$n[\mathit{in}\ m.P \mid Q] \mid m[R] \rightarrow m[n[P \mid Q] \mid R]$	(Red In)
$m[n[\mathit{out}\ m.P \mid Q] \mid R] \rightarrow n[P \mid Q] \mid m[R]$	(Red Out)
$\mathit{open}\ n.P \mid n[Q] \rightarrow P \mid Q$	(Red Open)
$(x_1, \ldots, x_n)P \mid \langle M_1, \ldots, M_n \rangle \rightarrow P\{x_i := M_i\}_{i=1}^{n}$	(Red Comm)
$P \rightarrow Q \Rightarrow (\nu n)P \rightarrow (\nu n)Q$	(Red Res)
$P \rightarrow Q \Rightarrow n[P] \rightarrow n[Q]$	(Red Amb)
$P \rightarrow Q \Rightarrow P \mid R \rightarrow Q \mid R$	(Red Par)
$P' \equiv P, P \rightarrow Q, Q \equiv Q' \Rightarrow P' \rightarrow Q'$	(Red $\equiv$)

Fig. 2. Operational Semantics of the Ambient Calculus

composition, provided the restricted name does not appear in the other components:

$$P \mid (\nu n)Q \equiv (\nu n)(P \mid Q) \quad \mathit{if}\ n \notin fn(P)$$

As said before, if process P above does have n as a free name and we want P and Q to interact we can always apply α-conversion. This can also be applied to ambients as rule (Struct Res Amb) shows.

The reduction rules mainly present the axioms for mobility and communication. Ambients can move into their sibling ambients or out of their enclosing ambient, as said in rules (Red In) and (Red Out) respectively. They may also dissolve the boundary of their subambients, so that the processes contained in the opened ambient now belong to the opener ambient, as defined in rule (Red Open). Finally, communication may happen inside them (Red Comm). The rest of the rules state that reductions may occur inside some constructors, namely restriction, ambients, and parallel, but not inside inputs, prefixes, or replications. Finally, rule (Red $\equiv$) makes explicit the fact that we are working modulo structural equivalence.

As an illustrative example of the semantics, let us consider the example

$$n[a[\mathit{out}\ n.\mathit{in}\ m.\langle M \rangle]] \mid m[\mathit{open}\ a.(x)Q]$$

This process can evolve in the following way:

$$\begin{array}{ll}
(n[a[\mathit{out}\ n.\mathit{in}\ m.\langle M\rangle]] \mid m[\mathit{open}\ a.(x)Q]) & \equiv \\
(n[a[\mathit{out}\ n.\mathit{in}\ m.\langle M\rangle] \mid 0] \mid m[\mathit{open}\ a.(x)Q]) & \rightarrow \\
a[\mathit{in}\ m.\langle M\rangle] \mid n[0] \mid m[\mathit{open}\ a.(x)Q] & \equiv (1) \\
n[0] \mid a[\mathit{in}\ m.\langle M\rangle \mid 0] \mid m[\mathit{open}\ a.(x)Q] & \rightarrow (2) \\
n[0] \mid m[a[\langle M\rangle \mid 0] \mid \mathit{open}\ a.(x)Q] & \equiv \\
n[0] \mid m[\mathit{open}\ a.(x)Q \mid a[\langle M\rangle]] & \rightarrow \\
n[0] \mid m[(x)Q \mid \langle M\rangle] & \rightarrow \\
n[0] \mid m[Q\{x := M\}] &
\end{array}$$

where, for instance, the equivalence (1) can be proved to hold using rules (Struct Par), (Struct Par Comm), (Stru Zero Par) and (Struct Amb), and step (2) can take place using the rules (Red Par) and (Red In).

4 An Implementation of Mobile Ambients in Maude

In this section we implement in Maude the operational semantics of AC. We have tried to be as faithful as possible to the way in which Cardelli describes the calculus. First, we define the syntax, and discuss about the mechanisms we need to manage bound names and variables. Then we implement operational semantics through both equations and rewrite rules. Finally we define strategies that control the application of rewrite rules. All the code is available in Maude's site [19].

4.1 Syntax definition

We define here AC syntax. For the sake of readability we omit variable declarations and operators precedence in most of the source code.

Syntax definition has to consider how to deal with bound names and variables. In AC there are two binding operators: the creation of new names (νn) that binds names and the input action (x) that binds variables. We need de Bruijn's indexes [2] in order to distinguish occurrences of the same name or variable that are bound by different binding operators. In order to illustrate this, see the following example:

$$(\nu n)(m[\mathit{open}\ k.(\langle n\rangle \mid n[0])] \mid k[\mathit{in}\ m.(x)(\nu n)n[\mathit{in}\ x.0]])$$

Ambient k moves into ambient m, there it is opened and name n is read, so that x is bound to (outer) name n. However such name n is different from the newly (inner) created name n so a renaming is mandatory. As potentially infinite names could be needed (due to replication), indexed names n_i can be used instead, where i represents the number of intermediate n-bindings between the free occurrence and its binding occurrence. In this way we can generate easily fresh names without losing the original ones. The same happens with variables. Initially the indexes are 0 and they are increased when α-conversion or communication (through substitution application) takes place.

The previous example would produce the following sequence of steps:

$$\begin{array}{ll}
(\nu n)(m_0[\mathit{open}\ k_0.(\langle n_0\rangle \mid n_0[0])] \mid k_0[\mathit{in}\ m_0.(x)(\nu n)n_0[\mathit{in}\ x_0.0]]) & \rightarrow \\
(\nu n)(m_0[\mathit{open}\ k_0.(\langle n_0\rangle \mid n_0[0]) \mid k_0[(x)(\nu n)n_0[\mathit{in}\ x_0.0]]]) & \rightarrow \\
(\nu n)(m_0[\langle n_0\rangle \mid n_0[0] \mid (x)(\nu n)n_0[\mathit{in}\ x_0.0]]) & \rightarrow (1) \\
(\nu n)(m_0[n_0[0] \mid (\nu n)n_0[\mathit{in}\ n_1.0]]) & \rightarrow (2) \\
(\nu n)(m_0[(\nu n)n_1[0] \mid n_0[\mathit{in}\ n_1.0]]) & \rightarrow \\
(\nu n)(m_0[(\nu n)n_1[n_0[0]]]) &
\end{array}$$

In steps (1) and (2) increasing of some indexes takes place due to communication and α-conversion respectively.

Consequently we have to use indexed names and variables in AC syntax. We can use Maude's `Qid` to define both. For the sake of clarity, names are `Qid`s beginning with letters 'a' to 'u', and variables those beginning with letters 'v' to 'z', which can easily be defined using membership axioms. Indexed names and variables, which we call `Acid`, are defined as:

```
sorts Qidn Qidx .
subsorts Qidn Qidx < Qid .

var q : Qid .
cmb q : Qidn if first-char(q) < "u" .
cmb q : Qidx if first-char(q) >= "u" .

sorts Name Var Acid .
subsorts Name Var < Acid .

op _{_} : Qidx Nat -> Var .
op _{_} : Qidn Nat -> Name .
```

Notice that in a system defined by a user that wants to execute an example, every name and variable has a 0 index, as indexes different from 0 only arise through communication or α-conversion. So we have defined a decoration function `dec` that fills the system defined by the user with the appropriate 0 indexes. We do not give here its definition (see [19] for the complete code).

Once defined names and variables, we can define straightforwardly messages and processes. Messages can be (indexed) names and variables, basic values (such as integers), capabilities, and paths:

```
sorts Message Capability Path .
subsorts Int Acid Capability Path < Message .

op in[_]   : Message -> Capability .
op out[_]  : Message -> Capability .
op open[_] : Message -> Capability .

op eps : -> Path .
op _._ : Message Message -> Path [assoc] .
```

In order to define processes we need to define first input and output sequences so that multiple communication can take place. Input sequences should rule out multiple occurrences of the same variable (this is done by defining the concatenation operator `_,_` for input sequences as partial `~>`, and giving a conditional membership that states when the concatenation is meaningful). Input sequences include type annotations (sort `AType`) for each input variable; this is because later we will define a type system for AC (for the moment they can be ignored).

```
sort InputSeq .
op _:_ : Qidx AType -> InputSeq .
op _,_ : InputSeq InputSeq ~> InputSeq [assoc] .

op bel : Qidx InputSeq -> Bool .
eq bel(x, y : T) = x == y .
eq bel(x, (I1, I2)) = bel(x, I1) or bel(x, I2) .
```

```
cmb (( x : T ), IS) : InputSeq if not bel(x, IS) .

sort OutputSeq .
subsorts Message < OutputSeq .
op _,_ : OutputSeq OutputSeq -> OutputSeq [assoc] .
```

Processes are defined as follows

```
sort Process .

op stop : -> Process .  *** 0 process
op _._ : Message Process -> Process .
op _|_ : Process Process -> Process [assoc comm id: stop] .
op !_ : Process -> Process .
op _[_] : Message Process -> Process .
op <_> : OutputSeq -> Process .
op (_)_ : InputSeq Process -> Process .
op new[_:_]_ : Qidn AType Process -> Process .
```

Using this syntax definition we can write the following (Cardelli's) firewall example:

```
ops P Q : -> Process .
eq firewall = new ['k : Amb[Shh]] ('n [open['k] . P]
 | new['m : Amb[Shh]] ('m ['k [ out['m] . in['n] . in['m] . stop] | Q])) .
```

where the type annotations can be ignored by now. Ambient `'m` can be regarded as a firewall that an agent `'n` wants to cross. The above mechanism can be used to guarantee authentication, to ensure freshness of messages by means of nonces or to model shared-key cryptography. By using process constants `P` and `Q` we will be able to universally quantify the execution of the process and give a general result for any two processes appearing there.

4.2 Substitutions and lifting

Now we discuss the functions we need to manage indexed names and variables [20]. In the previous example we have seen that indexes need to be increased some times, so several auxiliary definitions are necessary in order to modify de Bruijn's indexes. They can be defined as substitution constructors, so that later they can easily be combined with other substitutions. Substitutions are generated by communications.

First, we need a function that increases indexes when α-conversion takes place (step (2) in the example of Section 4.1):

```
op [shiftup_] : Qid -> Subst .
eq  [shiftup a] a{n} = a{s(n)} .
ceq [shiftup a] b{n} = b{n}  if a =/= b .
```

Now we have to manage communications and the substitutions arising from them. Simple substitutions replace variables by messages

```
sort Subst .
op [_:=_] : Qid Message  -> Subst .
```

In particular when applied, they replace only 0-indexed variables:

```
eq  [ a := M ] a{0} = M .
ceq [ a := M ] b{n} = b{n} if a =/= b .
```

We will never get to apply one simple substitution to a non zero variable as processes are closed with respect to variables.

We omit here the details about (standard) composition of substitutions.

Substitution application over messages and processes can be easily defined by cases. However we have to be careful when applied to processes with bound names or variables, as substitutions should be applied to the most external bound variable. In the following example

$$((x)(x)x_0[P] \mid x_1[Q]) \mid \langle n_0\rangle \rightarrow (3)$$
$$(x)x_0[P] \mid n_0[Q]$$

we want x_1, and not x_0, to be replaced by n_0.

This means that when we apply a substitution through a binding operator, we have to replace higher indexes than 0 (step (3) in the previous example). We will say that we *lift* the substitution:

```
op __ : Subst Process -> Process .
...
eq S (new[n : T]P) = new[n : T]([lift n S] P) .
eq S (I)P  = (I)(mlift(I, S) P) .
```

But also we also have to remember that free variables in the resulting process should be adequately shifted up to reflect the fact that there is one more level of intermediate binding (step (1) in the example of Section 4.1):

```
op [lift__] : Qid Subst -> Subst .

eq  [lift a S] a{0} = a{0} .
eq  [lift a S] a{s(n)} = [shiftup a] S a{n} .
ceq [lift a S] b{n} = [shiftup a] S b{n} if a =/= b .

op mlift : InputSeq Subst -> Subst .
eq mlift((x : T), S) = [ lift x S ] .
eq mlift(((x : T), I), S) = mlift(I, [lift x S]) .
```

Notice that `[lift a S]` when `S` is simple is a substitution that only affects variables indexed with 1, so first the indexes are decreased in order to apply `S` and then increased again. Subsequent nestings of liftings affect higher indexed variables if necessary.

Last operator `mlift` applies the `lift` operator to several variables. Multiple lifting can be sequencially applied due to the non-repeated variables constraint we have imposed over input variables.

4.3 Operational semantics

As we have previously seen, the operational semantics for AC consists of a set of structural congruence rules and a set of reduction rules.

Happily, Maude gives us some congruence rules for free. In particular:

– Rules (Struct Res) to (Struct Action) and (Struct Input), which define the congruence with respect to each process constructor, do not need to be defined due to equational congruence in Maude.
– Rules (Struct Par Assoc), (Struct Par Comm) and (Struct Zero Par) are obtained by indicating in the declaration of the parallel operator the associativity, commutativity, and identity attributes.

Rules (Struct ϵ), (Struct Path), (Struct Zero Res) and (Struct Zero Repl) are defined through Maude equations and consequently will be applied only from left to right. We write them looking for a *normal form* so that confluence holds:

```
eq eps . P = P .
eq (M . N) . P = (M . (N . P)) .

eq ! stop = stop .
eq new[n : T] stop = stop .
```

Extrusion rules (Struct Res Res), (Struct Res Par), and (Struct Res Amb) are written as three equations for α-conversion and (alphabetic) reordering of bound names looking for a normal form where all the bound names are at the top and ordered alphabetically:

```
ceq new[k : T1] new[l : T2] P = new[l : T2] new[k : T1] P if string(l) < string(k) .
ceq ((new[n : T]P) | Q) = new[n : T](P | ([shiftup n] Q)) if P =/= stop /\ Q =/= stop .
eq M [new[n : T] P] = new[n : T](([shiftup n] M)[P]) .
```

Notice that in order the first rule to be completely confluent we should define a (merely syntatic) order between the types in the (unfrequent) case the same name with different types is used. And then the reordering should rearrange the indexes of the involved names.

Notice also that the second (α-conversion) rule is only applicable when processes are different of `stop`. This condition will be necessary also several times below due to the identity attribute of `|` that could produce an infinite loop by generating once and again `| stop` in order to match with the equation. Another possibility would be to define a new type for non-stop processes and then write the equations or the rules with variables of such type.

We do not lose any power by writing the previous congruence rules as equations as they only reorder terms so that the subsequent reduction rules can be applied. For this to be true also parallel operator attributes and equational congruence are fundamental. The application of the equations produces a *normal form* where:

- `stop` only appears after a prefix (capability or input action) or inside an ambient;
- `eps` does not appear anywhere;
- sequences of capabilities associate to the right; and
- `new` operators are extruded as far as possible (in order interactions can take place) and are ordered alphabetically.

Rule (Struct Repl Par) will be discussed later. Finally we have the reduction rules as rewrite rules in Maude, some of which are conditional rewrite rules:

```
rl [RedIn] : n[in[m] . P | Q] | m[R] => m[n[P | Q] | R] .

rl [RedOut] : m[n[out[m] . P | Q] | R] => n[P | Q] | m[R] .

rl [RedOpen] : open[n] . P | n[Q] => P | Q .

rl [RedComm] : ((I)P) | < O > => bound(I,O) P .

crl [RedRes] : new[k : T] P => new[k : T] Q if P => Q .

crl [RedAmb] : n[P] => n[Q] if P => Q .

crl [RedPar] : P | R => Q | R if P =/= stop /\ R =/= stop /\ P => Q .
```

Function `bound(I,O)` generates a substitution as a result of the communication that is applied to the process, as we have previously explained.

Remember that the reduction relation of the calculus is not a congruence for all the operators, but only for restriction operator (Red Res), ambient construction (Red Amb), and parallel operator (Red Par). This means that we cannot freely use the rewrite rules we have written, as Maude would apply them anywhere in a term; and we do not want them to be applied after prefixes, inputs, and replication. This is one of the reasons why the definition of a strategy that controls the application of these rules is necessary.

Notice that we have the interleaving of congruence and reduction rules (Red $\equiv$) for free as Maude itself interleaves the application of equations with rewrite rules.

We study now what happens with replication. Replication behavior is described in AC through a congruence rule (Struct Repl Par). We cannot write it as an equation as the other ones because none of the orientations is convenient. If we apply it from left to right we get an infinite loop as we can infinitely unroll the replication. If we apply it from right to left we cannot see how the system evolves when new copies of the replicated process interact with other parts of the system or even with other copies of itself. As an example, in order the process $!n[in\ n.0]$ can evolve it is necessary to unroll replication twice:

$$\begin{array}{ll} !n[in\ n.0] & \equiv \\ n[in\ n.0]\ |!n[in\ n.0] & \equiv \\ n[in\ n.0]\ |\ n[in\ n.0]\ |!n[in\ n.0] & \rightarrow \\ n[in\ n.0|n[0]]\ |!n[in\ n.0] & \rightarrow \\ \ldots & \end{array}$$

This has led us to write this congruence rule as two rewrite rules:

```
rl [Rep] : ! P => P | ! P .
rl [UnRep] : P | ! P => ! P .
```

Still we have the same problem, so we have to define strategies to control the application of these rules. We want to apply rule `Rep` when it is necessary for subsequent interaction and rule `UnRep` to delete isolated unnecessary copies of the replicated process.

4.4 Strategies for evaluation

We need strategies to control the application of the rewrite rules defined before. Rules for movement and communication can be applied anywhere in the term but under prefixes and replication. So we define first a strategy to control the application of these rules called `norep` (no replication). As we have written rules for reducing inside ambients, in parallel processes, and under name restriction, we just have to apply all the rewrite rules at the top level. This means that the strategy will be applied recursively but that it will stop when a prefix or a replication is encountered.

Rules `RedRes`, `RedAmb` and `RedPar` are conditional rewrite rules so the strategy needs to know which strategy to apply in the rewrite condition and how to search in the resulting rewrite tree. In this case we want the same strategy to be (recursively) applied and a depth first search is enough for our purposes (strategies are defined in a `seq` declaration):

```
seq norep = top(RedIn) | top(RedOut) | top(RedOpen) | top(RedComm) |
            top(RedAmb{dfs(norep)}) |
            top(RedPar{dfs(norep)}) |
            top(RedRes{dfs(norep)}) .
```

So if we use `norep` strategy in the following examples

```
eq L = 'n{0} [in['n{0}] . stop] .
eq M =  L | L .
eq N = 'm{0} [M] .
eq P = ! M .
```

we obtain that:

- process `M` rewrites to `'n{0} [ in['n{0}] . stop | 'n{0} [ stop ] ]`,
- process `N` rewrites to `'m{0} ['n{0} [ in['n{0}] . stop | 'n{0} [ stop ] ]]`,
- process `P` cannot be rewritten.

Now we combine this strategy with a new one to control replication. We would like to unroll replication only when *necessary*: only when as a consequence of the unrolling a movement or a communication takes place. However we have to be careful because two unrollings could be *necessary* in order the movement or the communication takes place, as happened in process $!n[in\ n.0]$.

Additionally, even when one unrolling is enough to make a reduction step, we could lose rewrites if we force such reduction immediately. For example, if our strategy applied `norep` after each unrolling to process $n[0]\ |!n[in\ n.0]$, we could obtain the following rewriting:

$$\begin{array}{ll}
n[0] \mid !n[in\ n.0] & \equiv \\
n[0] \mid n[in\ n.0] \mid !n[in\ n.0] & \rightarrow \\
n[n[0]] \mid !n[in\ n.0] & \equiv \\
n[n[0]] \mid n[in\ n.0] \mid !n[in\ n.0] & \rightarrow \\
n[n[0] \mid n[0]] \mid !n[in\ n.0] & \\
\ldots &
\end{array}$$

so that only processes like $n[n[0] \mid \ldots \mid n[0]] \mid !n[in\ n.0]$ could be obtained, losing (among others) the following possible rewriting:

$$\begin{array}{ll}
n[0] \mid !n[in\ n.0] & \equiv \\
n[0] \mid n[in\ n.0] \mid !n[in\ n.0] & \equiv\ (1) \\
n[0] \mid n[in\ n.0] \mid n[in\ n.0] \mid !n[in\ n.0] & \rightarrow \\
n[0] \mid n[in\ n.0 \mid n[0]] \mid !n[in\ n.0] & \rightarrow \\
n[n[n[0]] \mid !n[in\ n.0] & \rightarrow \\
\ldots &
\end{array}$$

We claim that no more than two unrollings are necessary to obtain all the solutions, meaning by solutions those processes to which reduction rules cannot be applied any more. This can be easily proved by inspection of the rewriting trees for $S \mid P \mid P \mid !P$ and $S \mid P \mid P \mid P \mid !P$. The only difference is the level where we find the solutions.

Considering the two previous observations we define a new rule that allows us to unroll twice any replication appearing in the process but after prefixes and under replication (for the same reasons as `norep`)

```
rl [Rep2] : P => rep(P) .
```

being `rep` defined as

```
op  rep : Process -> Process .
eq  rep(! P) = P | P | ! P .
eq  rep(M[P]) = M[ rep(P) ] .
ceq rep(P | Q) = rep(P) | rep(Q) if P =/= stop and Q =/= stop .
eq  rep(new[n : T] P) = new[n : T] rep(P) .
eq  rep(P) = P [owise] .
```

As we want the unrolling to affect the whole process, this rule should be applied also at the top level

```
seq unroll-rep = top(Rep2) .
```

Of course, it can happen that unrolling does not help to the evolution of the process and just generates idle copies. In this case we apply rule `UnRep` to absorb those garbage copies. As an example, by unrolling twice and then communicating, process $\langle n\rangle \mid !(x)x[0]$ would rewrite to $n[0] \mid (x)x[0] \mid !(x)x[0]$ and then by applying rule `UnRep` we would obtain $n[0] \mid !(x)x[0]$.

Additionally, in order to avoid infinite computations when processes are nonterminating the user should tell the strategy how many real (Cardelli) reduction steps he wants to execute. Consequently, the strategy applies replication unrolling (if there is any) and immediately applies one more movement and/or communication step (if it is possible and we are not finished). When we are finished we eliminate every idle copy.

```
seq cardelli(0) = UnRep ! .
seq cardelli(s(n:Nat)) = (unroll-rep ; norep ; cardelli(n:Nat)) orelse (UnRep !) .
```

When rewriting process `! M` by using strategy `cardelli(1)` we obtain the following solution:

```
!('n{0}[in['n{0}]. stop]| 'n{0}[in['n{0}]. stop]) |
  'n{0}[in['n{0}]. stop | 'n{0}[stop]]
```

where one copy of `M` has evolved. When applying `cardelli(2)` we obtain the following three solutions:

```
Solution 1 :
  !('n{0}[in['n{0}]. stop] | 'n{0}[in['n{0}]. stop]) |
    'n{0}[in['n{0}]. stop] | 'n{0}[in['n{0}]. stop | 'n{0}[stop]| 'n{0}[stop]]
Solution 2 :
  !('n{0}[in['n{0}]. stop] | 'n{0}[in['n{0}]. stop]) |
    'n{0}[in['n{0}]. stop] | 'n{0}[in['n{0}]. stop | 'n{0}['n{0}[stop]]]
Solution 3 :
  !('n{0}[in['n{0}]. stop] | 'n{0}[in['n{0}]. stop])|
    'n{0}[in['n{0}]. stop | 'n{0}[stop]] | 'n{0}[in['n{0}]. stop | 'n{0}[stop]]
```

obtained by only two movements and one final application of `UnRep`. The three possibilities can be easily obtained by writing process $n[\mathit{in}\ n.0] \mid n[\mathit{in}\ n.0] \mid n[\mathit{in}\ n.0] \mid n[\mathit{in}\ n.0]$ and all the possible ways of making only two movements.

As a final example, when rewriting `firewall` using strategy `cardelli(4)` we obtain:

```
new['k : Amb[Shh]]new['m : Amb[Shh]]('m{0}[Q | 'n{0}[[shiftup 'm]P]])
```

as expected. Notice that as `P` and `Q` are just pseudo-processes there are some operations that cannot be applied like the shift-up and are left as such.

4.5 An Example: Electoral Systems

In [16] the problem of coding pure ambient calculus in π-calculus is studied. In particular, it is shown that symmetric electoral systems of arbitrary size exist for pure ambient calculus (AC with no communication), which implies that AC is not encodable in the π-calculus with separate choice as shown in [15]. The authors of [16] claim that the following process is a symmetric electoral system:

$$Net_k = P_0 \mid \ldots \mid P_{k-1}$$

$$P_i = n_i[\prod_{j \in S_i^k} in\ n_j.0 \mid \prod_{s \in T_i^k} m_i[in(s).out(s^-).out\ n_i.0]]$$

where $\prod$ denotes parallel composition, S_i^k is the set of all natural numbers less than k excluding i, T_i^k is the set of all strings of length $k-1$ using the members of S_i^k exactly once each, s^- is the string s in reverse order and $in(s)$ is the sequence of $in\ n_j$ for each successive $j \in s$ (respectively, $out(s)$).

For a symmetric net as the one above to be an electoral system it must be the case that all of its maximal computations produce exactly one *observable*, being all of them different. In this case, the observables are the ambients with names in $\{m_1, \ldots, m_{k-1}\}$ at the top level.

We have implemented the example above in our representation of Ambient Calculus. First, we define the functions `S : Nat Nat -> NatSet` and `T : Nat Nat -> NatListSet` to obtain the sets S_i^k and T_i^k, respectively. The sequence of capabilites $in(s)$ can be defined as follows:

```
op InList : NatList -> Path .

eq InList ( i IL ) = ( in[ 'n{i} ] . InList ( IL ) ) .
eq InList ( nil ) = eps .
```

We will use two auxiliary process definitions, `Pr1(S)` for process $\prod_{j \in S} in\ n_j.0$ and `Pr2(T,i)` for process $\prod_{s \in T} m_i[in(s).out(s^-).out\ n_i.0]$, in such a way that each P_i will be implemented by `'n{i}[ Pr1(S(i,k) | 'm{i}[ Pr2(T(i,k),i)] ] .`

```
op Pr1 : NatSet -> Process .
eq Pr1(i # NS) = ( in [ 'n{i} ] . stop ) | Pr1(NS) .
eq Pr1(mtNS) = stop .

op Pr2 : NatListSet Nat -> Process .
eq Pr2(IL ; ILS, i) = ('m{i}[ InList(IL) . (OutList(Rev(IL)) . out[ 'n{i} ] ) . stop])
                      | Pr2(ILS, i) .
eq Pr2(mt, i) = stop .
```

Then:

```
op Pr : Nat Nat -> Process .
eq Pr(i, k) = 'n{i}[ Pr1(S(i, k)) | Pr2(T(i, k), i) ] .
```

Finally, a net of size k is simply the parallel composition of the corresponding `Pr(i,k)`:

```
op Net : Nat -> Process .
eq Net(k) = elect(0, k) .

op elect : Nat Nat -> Process .
ceq elect(i, k) = Pr(i, k) | elect(i + 1, k) if i < k .
eq elect(k, k) = stop .
```

We can now take profit from our implementation of ambients to check that if we rewrite `Net(2)` using `cardelli(400)` we obtain:[1]

[1] We use 400 as a limit for the number of reduction steps in the strategy to make sure we obtain the maximal rewritings. This does not affect the efficiency.

```
Solution 1:
  'm{0}[stop] | 'n{0}[in['n{0}].stop | n{1}['m{1}[in['n{0}].out['n{0}].out['n{0}].stop]]

Solution 2:
  'm{1}[stop] | 'n{1}[in['n{1}].stop | n{0}['m{0}[in['n{1}].out['n{1}].out['n{1}].stop]]

No more solutions.
```

Indeed, there are only two possible (maximal) rewrites: solution 1 corresponds to observable `'m{0}` (n_0 wins) and solution 2 to observable `'m{1}` (n_1 wins). The same can be done with nets of size bigger than 3, getting analogous results.

5 A Type System for Mobile Ambients

In this section we first present Cardelli's type system for detecting communication errors. Then we define error reductions that precisely describe such errors and prove (by hand) that a well-typed process does not produce these communication errors along its execution (for more details see [18]). We implement the error reductions and define a strategy that allows us to know if a communication error occurs along the execution of a process. Then we implement the type system by using different techniques to manage the rules nondeterminism. Both implementations allow to infer the type of an annotated process as a result of the rewriting. Additionally, as a consequence of the study of the typing rules we have encountered that by adding a new rule, more processes that do not produce communication errors can be typed, and consequently we have slightly increased the power of the type system.

5.1 Types for the Ambient Calculus

In [7] the first type system for the Ambient Calculus is presented. Its main purpose is to avoid meaningless processes. Such processes may arise after some undesired communication interactions. For instance, the process

$$(x)x[P] \mid \langle n\rangle \mid (y)y.Q \mid \langle \mathit{open}\ n\rangle$$

may evolve to

$$n[P] \mid \mathit{open}\ n.Q$$

but also to

$$(\mathit{open}\ n)[P] \mid n.Q.$$

One way to avoid these meaningless terms[2] is to restrict the type of communications within each ambient, thus defining the *exchange types*. These types will not only specify whether ambients or capabilities are exchanged, but also what kind of ambients (what kind of information can be exchanged inside them) or what kind of capabilities (what kind of messages they unleash).

There are two kinds of exchange types: one for no exchange, *Shh*, and the other for tuple exchange, where each component will be an ambient type or a capability type, as shown in Figure 3.

The judgments of the type system are derived with respect to a type environment, as usually. Now we comment some of the typing rules shown in Figure 4:

[2] In fact they are only meaningless at the intuitive level. Formally they just include useless blocked subterms.

Exchange type	
$T ::= \mathit{Shh}$	no exchange
$\quad W_1 \times \ldots \times W_k$	tuple exchange
Message type	
$W ::= \mathit{Amb}[T]$	ambients that may contain exchanges of type T
$\quad \mathit{Cap}[T]$	capabilities that may unleash exchanges of type T

Fig. 3. Exchange Types

- (Zero) Process 0 does not produce any communication action. Thus, its natural type should be *Shh*. However, it can be understood that it has any type, so that if it is in parallel with any other process, it does not interfere with its communication behavior. Alternatively, it would be possible to introduce a subtype relation among types, giving 0 the minimal type, together with a new subsumption rule, as done in [24].
- (Amb) In order to type an ambient $M[P]$ one must check that its name is indeed an ambient name, $M : \mathit{Amb}[T]$. As process 0, it can be typed with any type.
- (In/Out) Movement capabilities do not unleash any exchange and, therefore, they can produce any capability type.
- (Open) If $M : \mathit{Amb}[T]$ then M is an ambient that contains processes of type T and, therefore, *open* M is a capability that may unleash exchanges of type T.
- (Prefix) This rule obliges P and $M.P$ to have the same type, which is the type determined by the prefix M when M is a capability *open* or a path containing one.
- (Parallel) Every sequential process within the same ambient must have the same type. This will only be a restriction for those processes that are responsible for communications.
- (Input) The residual of the input must be typeable with the same type that determines the input. Therefore, the communication type will be the same along the execution of the process.
- The rest of the rules are standard.

Rules (In/Out) and (Open), together with rule (Prefix) causes the opening capabilities to be the only ones that contribute to the type of a path. For example, if $\Gamma(n) = \mathit{Amb}[T]$ and $\Gamma(m) = \mathit{Amb}[S]$ then it holds that $\Gamma \vdash \mathit{in}\ n.\mathit{open}\ m : \mathit{Cap}[S]$.

5.2 Communication errors

In [7] only the intuitive meaning of types is described, but it is an interesting exercise to formalize it. In Figure 5 we define an error relation err_1. It can be considered to be a syntactic error, that arises from the use of two different kind of entities (names and capabilities) in the same syntactic category. Thus, an error is found, for instance, whenever a name is prefixing a process (instead of a capability). The definition of err_1 attempts to detect the error as soon as possible, in the sense that it looks in every subcomponent of the process, without considering variables, since we do not know what they will be replaced by.

If we suppose that $P \not\rightarrow \mathit{err}_1$ and $Q \not\rightarrow \mathit{err}_1$ then we can easily verify that:

(Exp n) $\quad \dfrac{\Gamma(n) = W}{\Gamma \vdash n : W}$

(Path) $\quad \dfrac{\Gamma \vdash M_i : \mathit{Cap}[T] \quad i = 1,2}{\Gamma \vdash M_1.M_2 : \mathit{Cap}[T]}$

(Empty) $\quad \dfrac{}{\Gamma \vdash \epsilon : \mathit{Cap}[T]}$

(In/Out) $\quad \dfrac{\Gamma \vdash M : \mathit{Amb}[T]}{\Gamma \vdash \mathit{in/out}\ M : \mathit{Cap}[S]}$

(Open) $\quad \dfrac{\Gamma \vdash M : \mathit{Amb}[T]}{\Gamma \vdash \mathit{open}\ M : \mathit{Cap}[T]}$

(Prefix) $\quad \dfrac{\Gamma \vdash M : \mathit{Cap}[T] \quad \Gamma \vdash P : T}{\Gamma \vdash M.P : T}$

(Amb) $\quad \dfrac{\Gamma \vdash M : \mathit{Amb}[T] \quad \Gamma \vdash P : T}{\Gamma \vdash M[P] : S}$

(Res) $\quad \dfrac{\Gamma, n : \mathit{Amb}[T] \vdash P : S}{\Gamma \vdash (\nu n : \mathit{Amb}[T])P : S}$

(Zero) $\quad \dfrac{}{\Gamma \vdash 0 : T}$

(Par) $\quad \dfrac{\Gamma \vdash P_i : T \quad i = 1,2}{\Gamma \vdash P_1 \mid P_2 : T}$

(Repl) $\quad \dfrac{\Gamma \vdash P : T}{\Gamma \vdash !P : T}$

(Input) $\quad \dfrac{\Gamma, x_1 : W_1, \ldots, x_k : W_k \vdash P : W_1 \times \ldots \times W_k}{\Gamma \vdash (x_1 : W_1, \ldots, x_k : W_k)P : W_1 \times \ldots \times W_k}$

(Output) $\quad \dfrac{\Gamma \vdash M_i : W_i \quad i = 1..k}{\Gamma \vdash \langle M_1, \ldots, M_k \rangle : W_1 \times \ldots \times W_k}$

Fig. 4. Typing rules for Exchange Types

$$\begin{tikzcd}
 & & n[P] \mid \mathit{open}\ n.Q \arrow[r, "/" marking] & err_1 \\
err_1 & (x : W)x[P] \mid \langle n \rangle \mid (y : W')y.Q \mid \langle \mathit{open}\ n \rangle \arrow[l, "/" marking] \arrow[ur] \arrow[dr] & & \\
 & & (\mathit{open}\ n)[P] \mid n.Q \arrow[r] & err_1
\end{tikzcd}$$

We have proved that typed processes do not cause such error. First we need an easy to prove lemma:

Lemma 1.

1. *If $\Gamma \vdash M : \mathit{Amb}[T]$ then $M \in \mathit{Id}$.*
2. *If $\Gamma \vdash M : \mathit{Cap}[T]$ then $M \in (\mathit{Cap} \cup \mathit{Var})^*$.*
3. *If $\Gamma \vdash M : W$ then $M \in \mathit{Id} \cup (\mathit{Cap} \cup \mathit{Var})^*$.*

and then we can prove the main theorem

Theorem 1. *If $\Gamma \vdash P : T$ then $P \not\to err_1$.*

Proof (sketch). This result can be proved by induction on the rules used to derivate $\Gamma \vdash P : T$ and using the previous lemma. Basically, it holds because processes that cause an error are those containing a subterm of the form $(cp\ N)[P]$, $\langle cp\ (cp'\ N) \rangle$, $n.P$, or $cp\ (cp'\ N).P$ (with $cp, cp' \in \{in, out, open\}$). These processes are not typeable, nor any process that contains them (in the type system every subterm must be typed in order to type the whole term).

$$\frac{M \notin (Var \cup Cap)^*}{M.P \rightarrow err_1} \qquad \frac{M \notin Id}{M[P] \rightarrow err_1} \qquad \frac{M_i \notin Id \cup (Cap \cup Var)^*}{\langle \tilde{M} \rangle \rightarrow err_1}$$

$$\frac{P \rightarrow err_1}{N[P] \rightarrow err_1} \qquad \frac{P \rightarrow err_1}{N.P \rightarrow err_1} \qquad \frac{P \rightarrow err_1}{(\tilde{x} : \tilde{W})P \rightarrow err_1}$$

$$\frac{P \rightarrow err_1}{(\nu n : W)P \rightarrow err_1} \qquad \frac{P \rightarrow err_1}{P \mid Q \rightarrow err_1} \qquad \frac{Q \rightarrow err_1}{P \mid Q \rightarrow err_1} \qquad \frac{P \rightarrow err_1}{!P \rightarrow err_1}$$

Fig. 5. Rules for syntactic errors

A subject reduction theorem for exchange types is proved in [7]. Using it we get our safety theorem:

Theorem 2. *If* $\Gamma \vdash P : T$ *and* $P \rightarrow^* Q$ *then* $Q \not\rightarrow err_1$.

It is straightforward to implement the error relation in Maude. For it we consider err_1 as a constant process `err1` and introduce rewritings from erroneous processes (according to the conditions stated in Figure 5) to `err1`.

```
op err1 : -> Process .

crl [errPref] : M . P => err1 if not isCap(M) .
crl [errAmb]  : M[P]  => err1 if not isAmb(M) .
crl [errMsg]  : < O > => err1 if not isMsg(O) .
```

The fact that errors are transmitted to the rest of the process is defined by the following equations, stating that any process containing an erroneous subterm is erroneous:

```
eq M[err1] = err1 .
ceq err1 | P = err1 if P =/= stop .
eq M . err1 = err1 .
eq ! err1 = err1 .
eq (I) err1 = err1 .
eq new[ n : T ] err1 = err1 .
```

Therefore, an error occurs whenever one of the three error rules above can be applied. The strategy `error1` tries to apply one of those rules. Then, `errcardelli` is a slight variation of the strategy `cardelli` in Section 4.4. It restricts normal steps to happen only when no error can be produced:

```
seq error1 = errPref | errAmb | errMsg .

seq errcardelli(0) = error1 orelse cardelli(0) .
seq errcardelli(s(n:Nat)) = error1 orelse (cardelli(1) ; errcardelli(n:Nat)) .
```

This strategy allows us to know if a given process produces sometime along its execution an `err1`. For example, the previous example written in Maude

```
eq fail = (('x : Amb[Shh]) ('x [P])) | < 'n > |
          (('y : Cap[Shh]) ('y . Q)) | < open['n] > .
```

rewritten with `errcardelli(3)` produces `P | Q` but also `err1`.

5.3 Two implementations of the type system

In order to implement the type system we first define the syntax for types. We have EType representing exchange types and MType representing message types. We also need TMType to represent tuples of messages types. We also have included a basic type for the integers bint.

```
sorts EType MType TMType .
subsorts MType < TMType < EType .

op bint : -> MType .
op Shh : -> EType .
op _x_ : TMType TMType -> TMType [assoc] .

op Amb[_] : EType -> MType .
op Cap[_] : EType -> MType .
```

Types decorate restricted names and input variables. When we defined ambients syntax, identifiers were decorated with (still not defined there) annotation types AType. As several type systems can be defined over the same syntax, we have decided to use AType as a supertype of any type that could annotate identifiers in a given type system. So when using a specific type system we have to say which types are used to annotate; here

```
subsort MType < AType .
```

Typing environments assign types to (indexed) names and variables. We have defined them over AType so that they can be used in other type systems. Their treatment is standard, so we only show here the operators syntax

```
sort  Env .
op empty : -> Env .                               *** empty env.
op __ : Env Env -> Env [assoc comm id: empty] .   *** envs. union
op (_,_) : Acid AType -> Env .                    *** type assignment

op _[_] : Env Acid -> AType .                     *** get the type of an id
op _[_->_] : Env Qid AType -> Env .               *** env. modification
op _[_] : Env InputSeq -> Env .                   *** multiple env. extension
```

We have to be careful when modifying an environment by extending it with a new type assignment: previously added assignments to the same identifier should be lifted up, so that we can identify the types of the different occurrences of identifiers with the same underlying string.

```
eq E[a -> T] = ([shiftup a] E) (a{0}, T) .
...
```

We now have to define the rules of the type system (Figure 4). We have written type judgements like $\Gamma \vdash P : T$ as rewrite rules $(\Gamma \vdash P) \longrightarrow T$ where a typing environment and a process are rewritten to the type of the process. In this way we infer the type as a result of the rewriting.

So we first define the lefthand sides (for processes and messages) of the typing rewrite rules:

```
sorts JudgeP JudgeM .
op _|-_ : Env Process -> JudgeP .
op _|-_ : Env OutputSeq -> JudgeM .
```

But in order to be able to rewrite terms of sort `JudgeP` to terms of sort `EType` these sorts have to belong to the same connected component (in the Maude subsort relation):

```
subsort EType < JudgeP .
subsort TMType < JudgeM .
```

As we have previously seen, type rules are highly nondeterministic. We would like to get inference by writing the rules as literally as possible. We study here two ways of treating nondeterminism in the rules.

The first implementation Some rules can be easily written as rewrite rules for typing messages:

```
rl [Exp]    : E |- a => E[a] .

crl [Tup]   : E |- M, O => W x TW if E |- M => W /\ E |- O => TW .

crl [Open] : E |- open[M] => Cap[T] if E |- M => Amb[T] .
```

and processes:

```
crl [Repl]    : E |- ! P => T if E |- P => T .

crl [Output] : E |- < O > => TW if E |- O => TW .

crl [Res]     : E |- new[n : Amb[T]] P => S if E[n -> Amb[T]] |- P => S .
```

Nondeterministic rules like (Zero) cannot be literally written as a rewrite rule, as we cannot rewrite to a partially undefined term. The same happens with rules (Empty), (In), and (Out) for typing messages. In fact, when we conclude that 0 has type T, we are saying that such T could be any type. Following this idea we define a new type constant `X` which means *any process type*:

```
op X : -> EType .
```

so that now we can write the following rules for messages

```
rl [Empty]  : E |- eps => Cap[X] .

crl [In]    : E |- in[M] => Cap[X] if E |- M => Amb[T] .

crl [Out]   : E |- out[M] => Cap[X] if E |- M => Amb[T] .
```

and for 0 process

```
rl [Zero] : E |- stop => X .
```

We still have to write rules for (Path), (Prefix), (Amb), (Par), and (Input). Let us study rule (Par), the rest of them are similar. Rule (Par) requires that the processes in parallel have the same type, so we could write:

```
crl [Par] : E |- P | Q => T
 if P =/= stop /\ Q =/= stop /\ E |- P => T /\ E |- Q => T .
```

but now we have a new type `X` that is any type and consequently that is compatible with any other one, so we need to add a new rule saying this:

```
crl [Par2] : E |- P | Q => T
 if P =/= stop /\ Q =/= stop /\ E |- P => T /\ E |- Q => X .
```

If any of the processes (or both) has type `X`, then they are compatible and the process can be typed. Due to commutativity we do not need to write a third rule.

The same happens with the rest of the rules; they are duplicated in order to consider the possible ways of compatibility: equality or typable with `X`. So, we have the rules for (Path):

```
crl [Path] : E |- M1 . M2 => Cap[T]
 if E |- M1 => Cap[T] /\ E |- M2 => Cap[T] .

crl [Path2] : E |- M1 . M2 => Cap[T]
 if E |- M1 => Cap[X] /\ E |- M2 => Cap[T] /\ T =/= X .

crl [Path3] : E |- M1 . M2 => Cap[T]
 if E |- M1 => Cap[T] /\ E |- M2 => Cap[X] /\ T =/= X .
```

Notice that in this case the operator is not commutative so we need three different versions of the rule.

For the rest of processes constructions we have the following rules:

```
crl [Prefix]  : E |- M . P => T if E |- M => Cap[T] /\ E |- P => T .
crl [Prefix2] : E |- M . P => T if E |- M => Cap[T] /\ E |- P => X .
crl [Prefix3] : E |- M . P => T if E |- M => Cap[X] /\ E |- P => T .

crl [Amb]  : E |- M[P] => X if E |- M => Amb[T] /\ E |- P => T .
crl [Amb2] : E |- M[P] => X if E |- M => Amb[T] /\ E |- P => X .

crl [Input]  : E |- (I) P => T
 if E[I] |- P => T /\ typeI(I) = T /\ T =/= X .
crl [Input2] : E |- (I) P => typeI(I) if E[I] |- P => X  .
```

While writing these rules we have noticed that rule (Open) is more restrictive than needed. If we try to type process $(\nu n : Amb[Shh])(open\ n.\langle n\rangle) \mid n[0])$, rule (Open) would give type $Cap[Shh]$ to $open\ n$ and consequently, rule (Prefix) could not be applied as $\langle n\rangle$ has type $Amb[Shh]$. However, when n is opened no communication error happens and the process just evolves to $(\nu n : Amb[Shh])\langle n\rangle$ with type $Amb[Shh]$.

The problem is that rule (Open) has not distinguished the case when the opened ambient has a silent type, like in the example. So we replace the previous rule `Open` by the following ones:

```
crl [OpenShh] : E |- open[M] => Cap[X] if E |- M => Amb[Shh] .
crl [Open]    : E |- open[M] => Cap[TW] if E |- M => Amb[TW] .
```

where the first one can only be applied to silent ambients and the other one to non-silent ambients.

The advantage of this form of implementation is that the rules are almost copies of the original rules being its disadvantage that in some cases they have to be duplicated. However, such duplication can be easily avoided by defining a *partial* function that computes the resulting type covering the different possibilities arising in the premises. For example, the rules `Par` and `Par2` would merge into the following rule

```
crl [Par] : E |- P | Q => T''
 if P =/= stop /\ Q =/= stop /\ E |- P => T /\ E |- Q => T' /\ T'' := compare(T,T') .
```

where the operation `compare` is defined as

```
op compare : EType EType ~> EType [comm] .
eq compare(T, T) = T .
eq compare(T, X) = T .
```

and the matching equation (`:=`) binds `T''` only when `compare(T,T')` is defined.

As an example, let us see the firewall example we saw in Section 4. In order to type it we need to give particular processes `P` and `Q`. If they were `stop` then given the following environment

```
op E : -> Env .
eq E = ('n{0}, Amb[Shh]) ('m{0}, Amb[Shh]) ('k{0}, Amb[Shh]) .
```

the rewriting of `E |- firewall` returns `X`, so it is well-typed and has any type.

The example shown in Section 4.5 does not engage in any communication and therefore, if no erroneous term appears at the beginning, nor will it appear after any number of steps. Indeed, if we define the environment giving every ambient silent type:

```
op EnvElec : Nat -> Env .

eq EnvElec(0) = empty .
eq EnvElec(s k) = ('n{k}, Amb[Shh]) ('m{k}, Amb[Shh]) EnvElec(k) .
```

then we can try to type `Net(k)` under environment `EnvElec(k)`. If we rewrite, for instance, `EnvElec(3) |- Net(3)` we get `X` as the only result.

The second implementation Rule (Zero) tells us than we can give process 0 any type: as it does not communicate anything it imposes no constraints on its context. Type *Shh* represents silence so we can think that this is the smallest type we could give to 0, although if necessary we could give it more complex types, e.g. if it is in parallel with a process communicating ambients. However, once we have given a non-silent type we have to maintain it. This implies that there is an implicit flat subtype relation in this type system, where *Shh* (resp. *Cap*[*Shh*]) can be seen as subtype of any other process type (resp. capability type), and the rest of them are only related with themselves.

In this approach nondeterministic rules will give the smallest types *Shh* and *Cap*[*Shh*], and when equal types are required (like in rule (Par)) a least upper bound is calculated. Such least upper bound only exists when applied to equal types or when one of them is *Shh*. For this reason we use a new type, called `errType`, arising when the least upper bound, calculated by `[]` is not defined:

```
op errType : -> EType .

op _[]_ : EType EType -> EType [comm].

eq Shh [] T = T .
eq T [] T = T .
ceq T [] S = errType if T =/= S .
eq errType [] T = errType .
```

Rules `Exp`, `Tup`, `Open`, `Repl`, `Output` and `Res` are exactly the same as before. The rest of the rules for messages are now

```
rl [Empty] : E |- eps => Cap[Shh] .

crl [Path] : E |- M1 . M2 => Cap[T [] S]
 if E |- M1 => Cap[T] /\ E |- M2 => Cap[S] .

crl [In]   : E |- in[M] => Cap[Shh] if E |- M => Amb[T] .

crl [Out]  : E |- out[M] => Cap[Shh] if E |- M => Amb[T] .

crl [OpenShh] : E |- open[M] => Cap[Shh] if E |- M => Amb[Shh] .
```

Rules `In` and `Out` give silent capabilities, but when concatenated to build a path they must be compatible, so a least upper bound is calculated in rule `Path`.

For processes we have:

```
rl [Zero]    : E |- stop => Shh .

crl [Par]    : E |- P | Q => T [] S
 if P =/= stop /\ Q =/= stop /\ E |- P => T /\ E |- Q => S .

crl [Prefix] : E |- M . P => T [] S
 if E |- M => Cap[T] /\ E |- P => S .

crl [Amb]    : E |- M [ P ] => Shh
 if E |- M => Amb[T] /\ E |- P => S /\ T [] S = T .

crl [Input]  : E |- (I) P => T
 if E[I] |- P => S /\ T := typeI(I) /\ S [] T = T .
```

With these rules the firewall example has type `Shh`, that is, we obtain the minimum type, from which the rest of them could be obtained: it can be proved that if a process has type *Shh* then it has also any other type T. The advantage of this implementation is that there is no duplication of rules and that if the process is not typable then it tells you so. For instance, process `fail` rewrites to `errType` in any type environment.

However if we want to obtain all the possible types like in the previous system we have to discover the implicit subtype relation (if there exists) which could not be obvious.

6 Conclusions and future work

We have exploited many features of the high-level language Maude in order to implement different semantics, both operational and static ones, for Cardelli's Ambient Calculus. First, we have implemented the operational semantics given by Cardelli in [4]. Although we follow the approach used in [23] of mapping reduction rules to rewrite rules, due to the particularities of the Ambient Calculus we have used the recently designed strategy language for Maude [14] in order to control the application of the rewrite rules. As far as we know this is the first time that this language is used to implement a calculus with mobility.

The treatment we have done of the replication operator by means of rules controlled by strategies is different from the approach used in [21] for the π-calculus. In the π-calculus the reduction rules are somewhat more compositional allowing the recursive definition of the replication operator. On the contrary, in the standard Ambient Calculus semantics there cannot be a compositional reduction rule for replication as there is a control flow from inner processes to the outside. For example, the evolution of $n[!in\ m.0] \mid m[0]$ is not defined in terms of the evolution of $!in\ m.0$, but instead making $!in\ m.0$ congruent to $in\ m.0 \mid !in\ m.0$, so that the movement can take place.

We have also implemented a type system for the Ambient Calculus defined in [7] to detect communication errors. There, only an intuitive meaning of the types was given. So first we have formally defined the errors intended to be captured by the type system and proved that well-typed processes do not produce such errors. From this result, together with Cardelli's subject reduction result, we can conclude that the type system is sound (more details in [18]). Then we have implemented the typing rules. These are highly nondeterministic. Usually nondeterminism in typing rules [17] arises due to the existence of several applicable rules to the same term or because different premises can be chosen in order to type the term. Such nondeterminism is treated to get type inference algorithms by modifying the rules or by applying the nondeterministic ones following a strategy (only at certain points of the type derivation). The nondeterminism arising in this system is quite different as, even thought the rules are completely syntax-directed and in this sense deterministic, the conclusions of the rules are not uniquely determined. We have shown two different ways of treating this nondeterminism and we have discussed the advantages and drawbacks of each one. Additionally, we have added a new typing rule that slightly strengths the power of the type system.

We are extending the work presented here to more sophisticated type systems like those defined in [5, 6]. We want to study if the same techniques can be applied to other calculus with mobility (AC variants) like for example Safe Ambients [12]. We also want to compare our results with existing inference algorithms like the one presented in [24]. In this sense it is our aim to go further and get type reconstruction, i.e. to infer also the type annotations needed (if any) to type an initially non-annotated process. For this purpose, we will have to introduce type variables in the type system and unifying mechanisms. We should also formalize some claims along the paper that have only been proved informally; for example the fact that two unrollings of a replicated process are enough to get all the possible rewritings.

We are studying how to extend the ITP tool, the inductive theorem prover for Maude, to allow proofs by induction on the rewrite rules. In its current state, the ITP allows to work with Maude equational specifications, proving properties by induction on terms. Induction on rules would allow us to prove, in a (semi)automatic way, properties like that the two given implementations of the type system are equivalent, or that typed processes do not produce errors.

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