

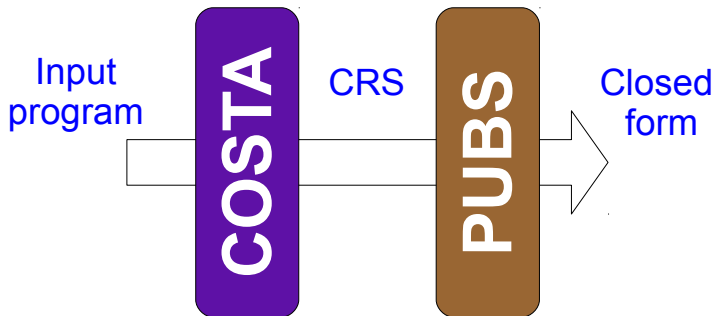
# Introduction to the COSTA Web Interface

Manuel Montenegro

Dpto. Sistemas Informáticos y Computación (DSIC)  
Facultad de Informática  
Universidad Complutense de Madrid (UCM)

Reliability of Software Systems  
Master Computing Science

# Overview of COSTA architecture



- A **Cost Relation System** (CRS) is a finite set of equations of the form:

$$C(\bar{x}) = \text{exp} + \sum_{i=1}^n C(\bar{y}_i) \quad \varphi$$

- The syntax of basic cost expressions is defined as follows:

$$\begin{aligned} \text{exp} ::= & r \mid \text{nat}(l) \mid \text{exp} + \text{exp} \mid \text{exp} * \text{exp} \mid \text{exp}^r \mid \log_n(\text{exp}) \\ & \mid n^{\text{exp}} \mid \text{max}(S) \mid \text{exp} - r \end{aligned}$$

where  $r \in \mathbb{Q}$ ,  $n \in \mathbb{N}$  and  $l$  is a linear expression:  $a_0 + a_1x_1 + \dots + a_nx_n$

# Syntax of PUBS declarations

$$C(\bar{x}) = \text{exp} + \sum_{i=1}^n C(\bar{y}_i) \quad \varphi$$

eq(Head, BasicExp, [RecCalls], [GuardConditions]).

# Syntax of PUBS declarations

$$C(\bar{x}) = \text{exp} + \sum_{i=1}^n C(\bar{y}_i) \quad \varphi$$

eq(Head, BasicExp, [RecCalls], [GuardConditions]).

- Head:  $c(X, Y, \dots, Z)$
- Basic expressions take the following syntax:

```
BasicExp ::= PosInt | nat(LinearExpression) | BasicExp * BasicExp
           | BasicExp + BasicExp | pow(PosInt, BasicExp)
           | log(PosInt, BasicExp) | max([BasicExp1, ...])
           | BasicExp / PosInt     | BasicExp - PosInt
```

- Examples of linear expressions:  $X+Y$ ,  $2*X$ ,  $X-3*Z$

# Syntax of PUBS declarations

$$C(\bar{x}) = \text{exp} + \sum_{i=1}^n C(\bar{y}_i) \quad \varphi$$

eq(Head, BasicExp, [RecCalls], [GuardConditions]).

- Each element of RecCalls has the same syntax as Head.
- $t(2 * X)$  can be translated into  $t(Y)$ , if we add the constraint  $Y = 2 * X$ .
- In some cases, inline expressions are allowed in the RecCall (just try!)

# Syntax of PUBS declarations

$$C(\bar{x}) = \text{exp} + \sum_{i=1}^n C(\bar{y}_i) \quad \varphi$$

eq(Head, BasicExp, [RecCalls], [GuardConditions]).

- **GuardConditions** is a comma-separated list of linear conditions:

LinearExpression >= LinearExpression

LinearExpression = LinearExpression

- $X > Y$  can be translated into  $X \geq Y + 1$
- $\frac{1}{2}X = Y$  can be translated into  $X = 2 * Y$

## Example

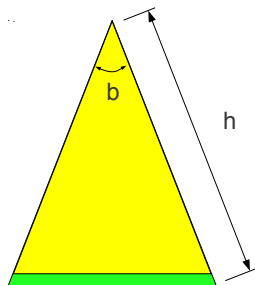
$$\begin{aligned} T(x,y) &= 0 && \{x \geq y\} \\ T(x,y) &= x^2 + T(x+1,y) && \{x < y\} \end{aligned}$$

`eq(t (X, Y), 0, [], [X>=Y]) .`

`eq(t (X, Y), nat (X) * nat (X), [t (X+1, Y)], [Y>=X+1]) .`



# Computing a closed form



$$C^+(\bar{x}) = nr(\bar{x}) * cr(\bar{x}) + nb(\bar{x}) * cb(\bar{x})$$

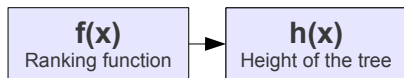
# Computing a closed form

**$f(\mathbf{x})$**

Ranking function

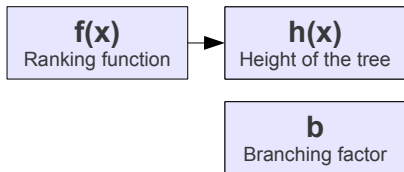
- Compute a ranking function  $f(\bar{x})$ 
  - Podelsky and Rybalchenko, 2004. (**linear!**)

# Computing a closed form



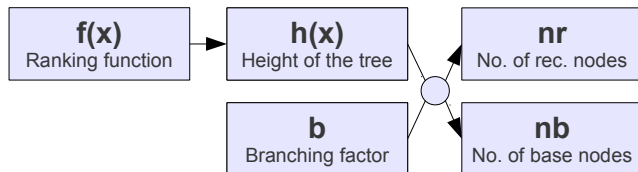
- Compute an upper-bound to the height of the call tree:
  - In the general case:  $h(\bar{x}) = f(\bar{x})$
  - Sometimes this bound can be further refined:  $\frac{f(\bar{x})}{\delta}$ ,  $\log_{\delta}(\bar{x})$

# Computing a closed form



- Determine the branching factor  $b$

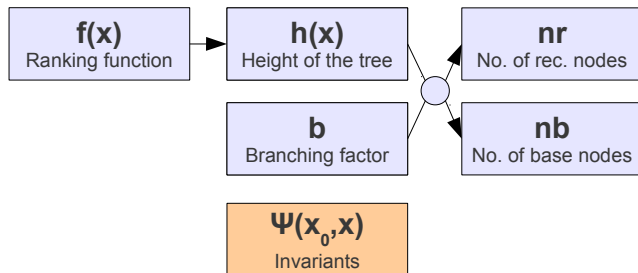
# Computing a closed form



- Compute an upper-bound to the number of base and recursive nodes  $nr(\bar{x})$ ,  $nb(\bar{x})$

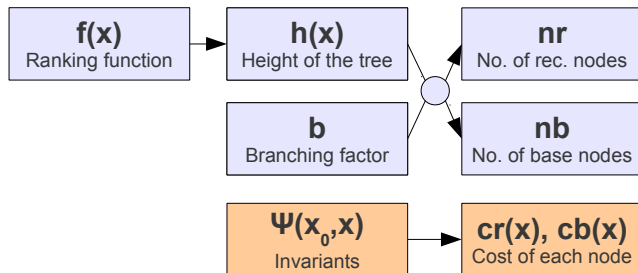
$$nb(\bar{x}) = b^{h(\bar{x})} \quad nr(\bar{x}) = \begin{cases} h(\bar{x}) & \text{if } b = 1 \\ \frac{b^{h(\bar{x})} - 1}{b - 1} & \text{if } b \geq 2 \end{cases}$$

# Computing a closed form



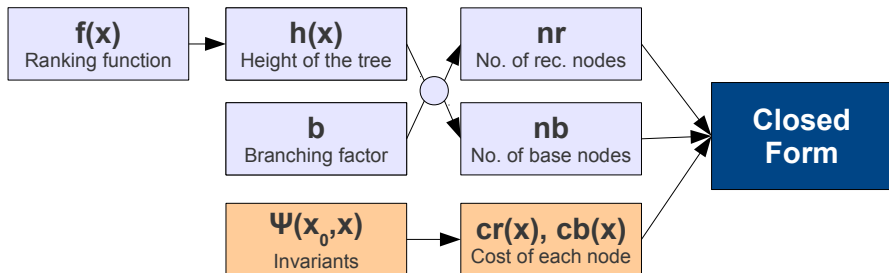
- Compute the relation  $\Psi$  between the root call and every child call.

# Computing a closed form



- Maximize each  $nat(l)$  component, by using the corresponding invariants.

# Computing a closed form



$$C^+(\bar{x}) = nr(\bar{x}) * cr(\bar{x}) + nb(\bar{x}) * cb(\bar{x})$$

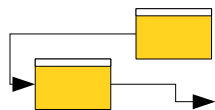
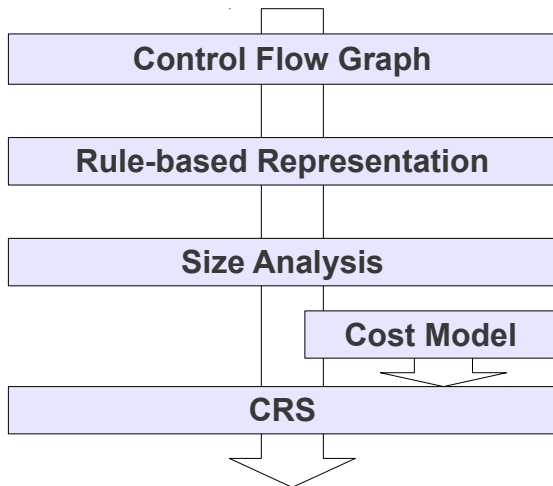


## Example

$$\begin{aligned} T(x) &= 0 && \{x \leq 0\} \\ T(x) &= 1 + T(x') + T(x'') && \{x > 0, x > 2 * x', x > 2 * x''\} \end{aligned}$$

$\text{eq}(t(X), 0, [], [0 \geq X])$ .

$\text{eq}(t(X), 1, [t(X1), t(X2)], [X \geq 1, X \geq 2 * X1 + 1, X \geq 2 * X2])$ .



$p(x) \leftarrow \dots q(y) \dots$

$y = 2 * x$

# COSTA Example (I)

```
public void method1(int n, int m) {  
    int[] array;  
    for (int i = 0; i < n; i++) {  
        array = new int[m];  
    }  
}
```

## COSTA Example (II)

```
public static final int ARRAY_SIZE = 20;

public void method2(int n) {
    int[] array;
    for (int i = 0; i < n; i++) {
        array = new int[ARRAY_SIZE];
    }
}
```

## COSTA Example (III)

```
public void method3(int n, int m) {  
    int numIterations = n * m;  
    for (int i = 0; i < numIterations; i++) {  
        System.out.println("Iteration");  
    }  
}
```

## COSTA Example (III)

```
public void method3(int n, int m) {  
    int numIterations = n * m;  
    for (int i = 0; i < numIterations; i++) {  
        System.out.println("Iteration");  
    }  
}
```

- The size of `numIterations` does not depend **linearly** on `n` and `m`.

- **The COSTA System:**  
`https://costa.ls.fi.upm.es/`
- **PUBS:**  
`http://costa.ls.fi.upm.es/~costa/pubs/pubs.php`
- **COSTA:**  
`http://costa.ls.fi.upm.es/~costa/costa/costa.php`



E. Albert, P. Arenas, S. Genaim, M. Gómez-Zamalloa, G. Puebla, D. Ramírez, G. Román, D. Zanardini  
*Termination and Cost Analysis with COSTA and its User Interfaces.*  
Spanish Conference on Computer Languages and Programming  
(PROLE 2009), ENTCS, Elsevier.