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Patterns

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1. Motivation

2. Design space/decisions

3. Some λ-calculi with patterns

4. Patterns in Haskell

Motivation

Conceptual

- λ-calculus: parameter passing rule $\beta : (\lambda x.M)N \to M[x := N]$

$$(\lambda x\, y.x + y)\, 0\, 3 \to (\lambda y.0 + y)\, 3 \to 0 + 3$$

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  $$0 + 3 \to 3$$

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  $0 + 3 \to 3$
- λ-calculi with patterns combine parameter passing with pattern matching

  $(\lambda[x, y].x)\, [1, 2, 3, 4, 5] \to 1 \quad \text{and} \quad (\lambda[x, y].y)\, [1, 2, 3, 4, 5] \to [2, 3, 4, 5]$

  Haskell-like list $[1, 2, 3, 4, 5]$ abbreviates $[1, [2, [3, [4, [5]]]]]$; repeated cons

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in math common practice to write $(x, y) \mapsto x + y$ for addition function on pairs
a pair (x, y) is already a rudimentary form of a pattern

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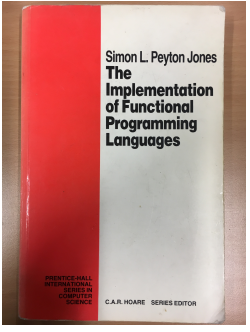
This lecture

Trying to make sense of this idea, of λ-calculi with patterns

Motivation

Personal (1990)

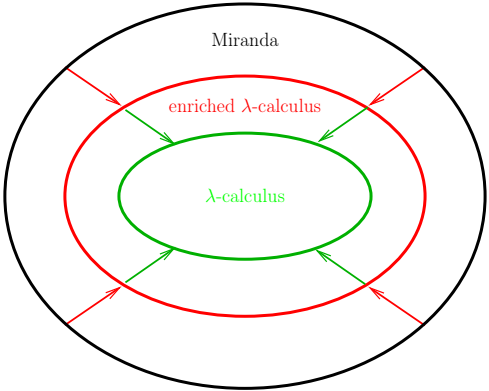
Klop (PhD supervisor) to Vincent (PhD student): the enriched λ-calculus in the recent book [SPJ] sounds interesting, could you study its rewrite properties?



How to implement functional programming languages?

Idea of [SPJ]: transformational

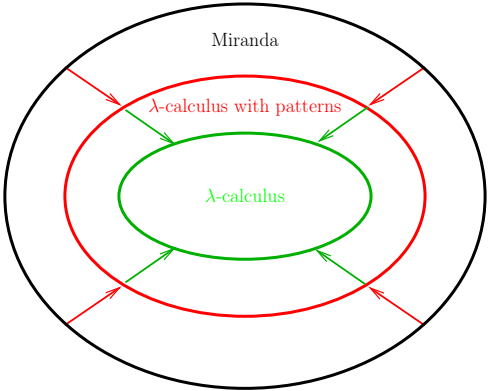
FPL $\Longrightarrow$ enriched λ-calculus (Chapter 4) $\Longrightarrow$ λ-calculus (with []; Chapter 6)



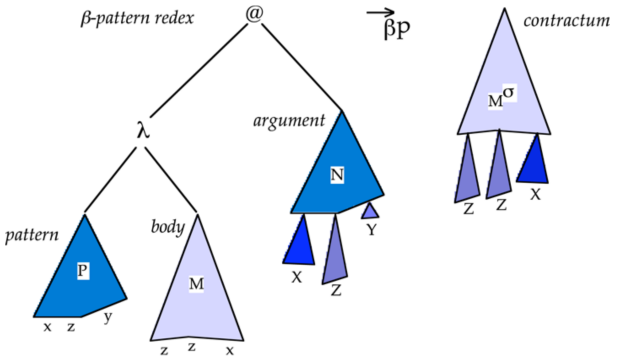
How to implement functional programming languages?

Idea of [SPJ]: transformational

FPL $\Longrightarrow$ λ-calculus with patterns (Chapter 4) $\Longrightarrow$ λ-calculus (with []; Chapter 6)



Pattern reduction in a picture [KvOdV]

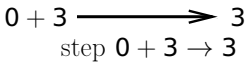


Idea

if argument is a substitution σ ($x \mapsto X$, $z \mapsto Z$, $y \mapsto Y$) instance N^σ of the pattern ($P = N$), then contractum M^σ is substitution instance of body M

Design space

The three stages of reduction (for rule $0 + x \to x$)



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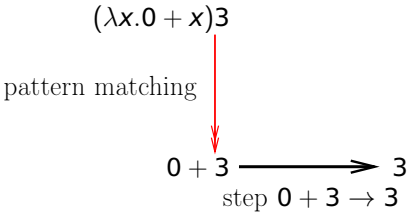
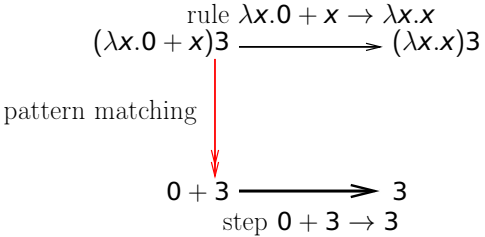


Design space

The three stages of reduction (for rule $\lambda x.0 + x \to \lambda x.x$)

1. matching: matching and abstracting lhs ℓ of rule $\ell \to r$
2. replacement: replace ℓ by r (closure: $\forall \vec{x}.(\ell \to r) \implies (\lambda \vec{x}.\ell) \to (\lambda \vec{x}.r)$; Frege)



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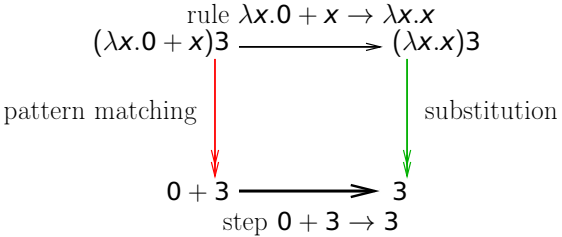

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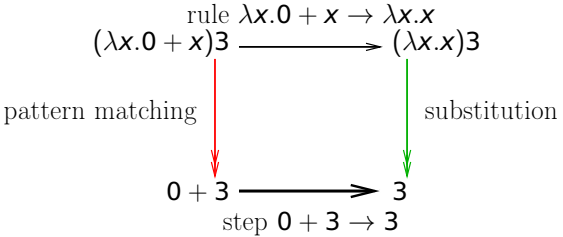

analogy = generalisation + specialisation (Pólya's △)

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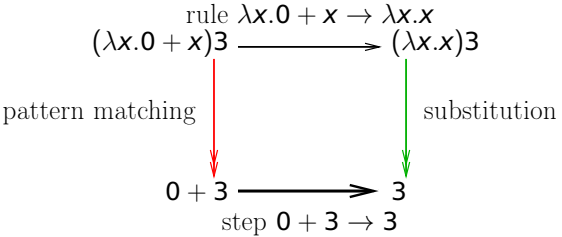

substitution calculus ($\lambda\alpha\beta\eta^{\to}$) = matching + substitution (van Raamsdonk, vO)

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λ-calculi with patterns design space

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- represent partial functions (λ) or anything goes (TRS)?

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Design decisions

The three stages of reduction (for rule $\lambda x.0 + x \to \lambda x.x$)

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- how to handle match failures? (wait? error if stably fails?)
- untyped or typed, fixed or arbitrary signature, ...

Design

The three stages of reduction (for rule $\lambda x.0 + x \to \lambda x.x$)

1. matching: matching and abstracting lhs ℓ of rule $\ell \to r$
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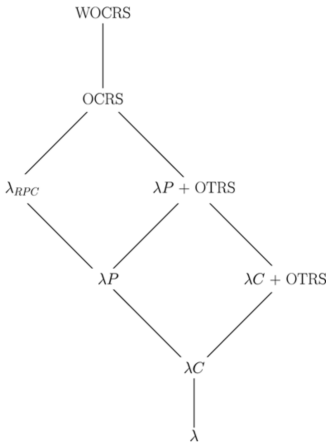
This lecture/hour

- represent (partial) functions
- implicit, syntactic matching and substitution (possibly higher-order)
- first just wait then with errors
- untyped and both arbitrary (λC and CRS/HRS) and fixed signatures (λ_{RPC})

λ-calculi with patterns of [KvOdV]



Lattice of λ-calculi with patterns

WOCRS: weakly ortho 2nd order TRS
OCRS: orthogonal 2nd/higher order TRS
λ_{RPC} : λP with rigid pattern condition
$\lambda P +$ OTRS: $\lambda P +$ orthogonal TRS
λP: $\lambda +$ pure patterns
$\lambda C +$ OTRS: $\lambda C +$ orthogonal TRS
λC: $\lambda +$ constructor patterns
λ: ordinary $\lambda\beta$-calculus

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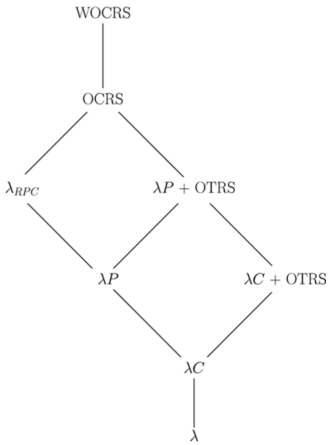

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λC, λ-calculus with constructor patterns (wait)

Idea

terms over an arbitrary first-order signature of constructor symbols as patterns

λC, λ-calculus with constructor patterns (wait)

Definition (λC for C arbitrary first-order signature)

$$P ::= x \mid c(\vec{P}^n) \qquad x \text{ a variable, } c \in C \text{ and } n\text{-ary}$$

$$M ::= P \mid \lambda P.M \mid M\,M \mid c(\vec{M}^n)$$

$$(\lambda P.M)P^\sigma \to M^\sigma \qquad \text{vars in } P \text{ bound in } M \text{ in } \lambda P.M$$

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Example (Simple list projection functions for $C = \{\mathrm{CONS}/2, A/0\}$)

$$\lambda \mathrm{CONS}(x,y).x$$
$$\lambda \mathrm{CONS}(x,y).y$$
$$(\lambda \mathrm{CONS}(x,y).y)\,\mathrm{CONS}(A,z) \to z$$

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(Dis)advantages

- compared to λ-calculus: express (free) data types without coding

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(Dis)advantages

- compared to λ-calculus: express (free) data types without coding
- compared to TRS: anonymous functions on data-types (see example)

Non-linearity breaks representing partial functions

Example (Klop's non-overlapping but non-confluent TRS $\{P/2, E/0\}$)

$$D = \lambda P(x,x).E$$
$$C = \Theta\, \lambda cx.D\, P(x, c\, x)$$
$$A = \Theta\, C$$

Θ is Turing's fixed point combinator: $\Theta\, M \twoheadrightarrow M\, (\Theta\, M)$ for any term M

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Remark (Imitating Turing (Klop))

How to build your own fixed-point combinator in three easy steps:

- *assume your fpc has repetitive shape $A\,A$ for some A*
  (idea: left A is the actor, right A is the passor)

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- *assume fp-behaviour is exhibited by expansion $A\,A\,M \twoheadrightarrow M\,(A\,A\,M)$*
- *conclude may set $A = \lambda am.m\,(a\,a\,m)$. Indeed $\Theta = AA$.*

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$$\begin{array}{ccccccccc} A & \twoheadrightarrow & CA & \twoheadrightarrow & DP(A,CA) & \twoheadrightarrow & DP(CA,CA) & \to & E \\ \downarrow & & & & & & & & \\ CA & \twoheadrightarrow & C(CA) & \twoheadrightarrow & C(DP(A,CA)) & \twoheadrightarrow & C(DP(CA,CA)) & \to & CE \end{array}$$

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Comment on non-joinability of E and $C\,E$

if $C\,E$ and E were joinable $\implies C\,E \twoheadrightarrow E \implies C\,E \twoheadrightarrow D\,P(E, C\,E) \twoheadrightarrow E$

Non-linear non-overlapping TRSs not representable

Example (Huet's non-overlapping but non-confluent TRS)

$$\infty \to S(\infty)$$
$$E(x,x) \to \mathit{true}$$
$$E(x,S(x)) \to \mathit{false}$$

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$$E(\infty,\infty) \to \mathit{true}$$
$$\downarrow$$
$$E(\infty,S(\infty)) \to \mathit{false}$$

Comment on non-definability in λC

true and *false* are distinct normal forms, but λC has the unique normal form property (also if non-linear patterns are allowed).

Some λC rewriting meta-theory

Theorem (Linear λC represents partial functions)

λC is confluent if patterns are required to be linear

Some λC rewriting meta-theory

Theorem (Linear λC represents partial functions)

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Proof.

boring/wasteful: adapting the Tait–Martin-Löf proof □

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Proof.

better/reuse: linear λC embeds into orthogonal HRS (use Nipkow) □

Some λC rewriting meta-theory

Theorem (Linear λC represents partial functions)

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Theorem

λC has the unique normal form property
(normal forms N_1, N_2 are convertible $N_1 \leftrightarrow^* N_2 \implies N_1 = N_2$ the same)

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Proof.

boring/wasteful: adapting de Vrijer's proof □

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Theorem (Linear λC represents partial functions)

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Theorem

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Proof.

better/reuse: λC embeds in strongly non-overlapping HRS (Mano,Ogawa) idea:
N_1, N_2 minimal distinct normal forms with $N_1 \leftrightarrow^* N_2 \implies$
conditional linearizations (cl; de Vrijer) N_1^*, N_2^* are cl-convertible $\implies$
these are cl-joinable $\implies$
one of N_1^*, N_2^* is cl-reducible $\implies$
one of them has distinct convertible normal forms as subterms. contradiction $\square$

Some λC rewriting meta-theory

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Theorem

λC has the unique normal form property

Example (of waiting and its solution [SPJ])

for NIL/0 a nullary constant, the term $(\lambda \mathrm{CONS}(x, y).y)\,\mathrm{NIL}$ is stuck; it waits until the NIL-argument matches the CONS-pattern which will never happen. this can be observed in λC because constructors are distinct $\Longrightarrow$ adjoin case construct

λC, λ-calculus with constructor patterns (case)

Definition (λC for C arbitrary first-order signature)

$$P ::= x \mid c(\vec{P}^n) \qquad x \text{ a variable, } c \in C \text{ and } n\text{-ary}$$

$$M ::= P \mid \lambda P.M \;[]\; \ldots \;[]\; P.M \mid M\,M \mid c(\vec{M}^n)$$

$$(\lambda P_1.M_1 \;[]\; \ldots \;[]\; P_n.M_n)P_i^\sigma \to M_i^\sigma$$

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Example (Simple list projection functions for $C = \{\mathrm{CONS}/2, A/0, \mathrm{NIL}/0\}$)

$$(\lambda \mathrm{CONS}(x,y).y \,[]\, \mathrm{NIL}.A)\,\mathrm{NIL} \to A$$

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$$(\lambda P_1.M_1 \,[]\, \ldots \,[]\, P_n.M_n)P_i^\sigma \to M_i^\sigma$$

Example (Non-ortho patterns do not represent partial functions)

$$\infty = \Theta\, \lambda i.S(i)$$

$$E = \lambda P(x,x).true \,[]\, P(x,S(x)).false$$

$$N = (\lambda A.false \,[]\, A.true)\,A$$

∞ and E represent Huet's non-linear example; N has overlapping patterns

λC, λ-calculus with constructor patterns (case)

Definition (λC for C arbitrary first-order signature)

$$P ::= x \mid c(\vec{P}^n) \qquad x \text{ a variable, } c \in C \text{ and } n\text{-ary}$$

$$M ::= P \mid \lambda P.M \;[]\; \ldots \;[]\; P.M \mid M\,M \mid c(\vec{M}^n)$$

$$(\lambda P_1.M_1 \;[]\; \ldots \;[]\; P_n.M_n)P_i^\sigma \to M_i^\sigma$$

Theorem (Representing partial functions in λC with cases)

λC is confluent if patterns are required to be linear and pairwise non-unifiable

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Remark

can be extended with error rule in case argument is instance of (linear) pattern not unifiable with case-patterns. [SPJ]/Haskell use [] for sequential matching.

Gustave's TRS

Exercise

TRS with signature $\{G/3, \top/0, \bot/0, 1/0, 2/0, 3/0\}$ *and three rules*

$$G(\top, \bot, x) \to 1 \qquad G(\bot, x, \top) \to 2 \qquad G(x, \top, \bot) \to 3$$

and let $\Omega = (\lambda x.x\,x)\,(\lambda x.x\,x)$ *as usual*

- *is the TRS orthogonal (left-linear and non-overlapping/lhss pairwise non-unifiable)?*
- *give a* λC*-expression* D*, where all symbols in the above signature are now constructors, that exhibits the behaviour as specified, i.e.* $D\,G(\top, \bot, \Omega) \twoheadrightarrow 1$*,* $D\,G(\bot, \Omega, \top) \twoheadrightarrow 2$*, and* $D\,G(\Omega, \top, \bot) \twoheadrightarrow 3$
- *can you express this in Haskell (thinking of* $\top, \bot$ *as booleans, say) (for 2nd part but you can think about it already)*

Idea

have λ-terms themselves as patterns
no extension of signature; minimal calculus in spirit of beginning of 20th century

Definition (Provisional minimal syntax (λP))

$$M, P ::= x \mid \lambda P.M \mid M\,M \qquad x \text{ a variable; free vars of } P \text{ bound in } M \text{ in } \lambda P.M$$

$$(\lambda P.M)P^{\sigma} \to M^{\sigma}$$

λ_{RPC}

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Example (Simple list projection function in λ_{RPC})

$$(\lambda(\lambda z.z\,x\,y).x)\,\lambda z.z\,M\,N \to M$$
$$(\lambda(\lambda z.z\,x\,y).y)\,\lambda z.z\,M\,N \to N$$

$\lambda z.z\,M\,N$ is standard representation of a pair (M, N) in untyped λ-calculus

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- if variable is erased variables may be freed: $\lambda(\lambda x.y)z.z \to \lambda y.z$??

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- if variable is erased variables may be freed: $\lambda(\lambda x.y)z.z \to \lambda y.z$??
- applicative patterns give rise to inconsistency: $(\lambda(x\,y).x)\,I\,(K\,z)$ reduces in one step to I and in two steps to K?? for $I = \lambda x.x$ and $K = \lambda x.\lambda y.x$ as usual

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Definition (Restricting patterns P)

λ_{RPC} restricts patterns in abstractions to RPC comprising ordinary λ-terms that

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Theorem

λ_{RPC} is confluent

The case for higher-order term rewriting; CRSs/HRSs

Solution for ?

combinatory Logic (Schönfinkel) : first-order term rewrite systems

$=$

λ-calculus (Church) : ?

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Universal algebra approach to CL,λ-calculus

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$(\lambda\,[x,y].x)\,[1,2,3,4,5] \Longrightarrow \lambda\mathsf{RULE}([x,y],x)\,[1,2,3,4,5]$

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$@(\lambda x\, y.\text{RULE}([x, y], x), [1, 2, 3, 4, 5]) \to 1$ (with $F = \lambda ab.a$, $X = 1$, $Y = [2, 3, 4, 5]$)
$@(\lambda x\, y.\text{RULE}([x, y], F(x, y)), [X, Y]) \to F(X, Y)$

Embedding yields orthogonal HRS

Theorem

translation yields a (potentially infinite,weakly) orthogonal HRS
(if patterns in λC or λ_{RPC} are restricted to linear ones)

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HRSs may capture stages of transformation from FPL into assembly (CRSX, Rose)

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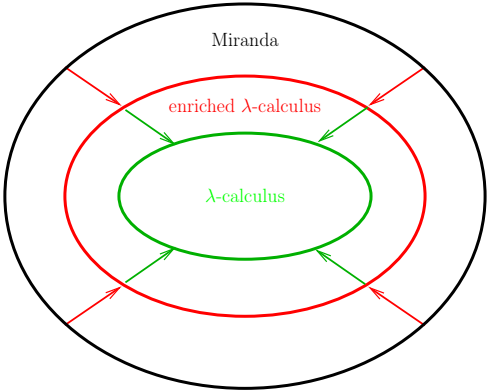
but always from a higher-order perspective

Recall implementing functional languages in [SPJ]

Idea of [SPJ]: transformational

FPL $\Longrightarrow$ enriched λ-calculus (Chapter 4) $\Longrightarrow$ λ-calculus (with []; Chapter 6)



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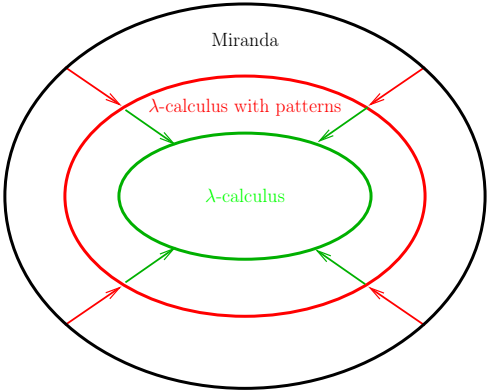

Compiling pattern matching into λ-calculus for Haskell

Idea

sequentialise matching symbol by symbol [SPJ] (Chapter 5, Wadler)

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Example (Remove repetitions of elements by pattern matching)

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unrep [] = []
unrep [x] = [x]
unrep (x:y:xs) = if x == y then unreptail else x:unreptail where
  unreptail = unrep (y:xs)
```

patterns match top–bottom (left–right); may have nested patterns like `x:y:xs` compile away using case

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- proper case split due to non-ambiguity and linearity of patterns (Haskell)
- case implementable in λ-calculus by discriminating constructors (Böhm)

Lazy patterns in Haskell

Motivation

sometimes we know, say due to typing or due to knowing that we have an infinite stream, that only one pattern is possible, say a pair, but we want to match lazily, e.g. we are not interested in the arguments, or only want to match when the arguments/projections are needed

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Example of pattern binding

```
fib@(1:tfib)     = 1 : 1 : [ a+b | (a,b) <- zip fib tfib ]
```

where `@` makes `fib` an alias of the pattern binding `1:tfib`

Lazy patterns in Haskell

Motivation

sometimes we know, say due to typing or due to knowing that we have an infinite stream, that only one pattern is possible, say a pair, but we want to match lazily, e.g. we are not interested in the arguments, or only want to match when the arguments/projections are needed
Haskell has lazy patterns; indicated by ~ in Haskell or pattern bindings

Example of pattern binding

```
fib@(1:tfib)    = 1 : 1 : [ a+b | (a,b) <- zip fib tfib ]
```

where `@` makes `fib` an alias of the pattern binding `1:tfib`

lazy patterns akin to speculative execution (bad things may happen)

Lazy patterns in Haskell translation exercise

Exercise on translating (lazy) patterns into cases

how does `ispair mypair` for the code below evaluate and why? same question for `ispairl` and `ispairw`. can your translate these definitions into case expressions/projections? what about when changing each `True` into `x+y`?

```
ispair (x,y) = True
ispairl ~(x,y) = True
ispairw p = True where
  x = fst p
  y = snd p
mypair = mypair
```

Note `(,)` is the only constructor of the pair data type

Lazy patterns in Haskell exercise

Exercise on lazy patterns

consider the following core client–server program; from Haskell [tutorial]:

```
reqs  = client initmsg resps
resps = server reqs
client msg (r:rs) = msg : client (next r) rs
server (r:rs) = process r : server rs
initmsg = 0
next r = r
process r = r+1
```

for this program `take 10 reqs` does not generate any output, but yields `[0,1,2,3,4,5,6,7,8,9]` when making `client` lazy by means of `~(r:rs)`.

can you explain this, based on sequentialising patterns as presented?